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EMPLOYING GENERALIZED MEIR-KEELER CONTRACTION FOR COINCIDENCE POINT RESULTS ON ORDERED METRIC SPACES WITH APPLICATION

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Abstract. We present coincidence point theorem for g-non-decreasing mappings satisfying generalized Meir-Keeler contraction on ordered metric spaces. Using obtained result, we demonstrate the formation of coupled coincidence point result for generalized compatible pair of mappings. We obtain the solution of Fredholm nonlinear integral equation to indicate the usefulness of our result and also give an example. Our results generalize, modify, improve and enrich several known results.

Keywords: Coincidence point, coupled coincidence point, generalized Meir-Keeler contraction, ordered metric space, O-compatible, generalized compatibility, g-non-decreasing mapping, mixed monotone mapping, commuting mapping.

AMS Subject Classification: 47H10, 54H25.

1. Introduction

In 1987, Guo and Lakshmikantham [10] introduced the notion of coupled fixed point for single-valued mappings. Pursuing this paper, Bhaskar and Lakshmikantham [4] constructed some coupled fixed point theorems on ordered metric spaces, by giving the concept of mixed monotone property. After that, Lakshmikantham and Ćirić [21] extended the notion of mixed monotone property to mixed g-monotone property and established coupled coincidence point results using a pair of commutative mappings, which generalized the results of Bhaskar and Lakshmikantham [4]. Succeeding it, in view of coupled coincidence point, the notion of compatibility was introduced by Choudhury and Kundu [5], thenafter they improve the results of Lakshmikantham and Ćirić [21] by using this notion. As an application, these results used to study the existence and uniqueness of a solution for a periodic boundary value problem associated with a first order ordinary differential equation.

Hussain et al. [12] introduced a new concept of generalized compatibility of a pair of mappings $F, G:X^2\to X$ and proved some coupled coincidence point results. Subsequently, Erhan et al. [8], indicated that the results established in Hussain et al. [12] can be derived from the coincidence point results in the existing literature.

On the other hand, Samet et al. [37] claimed that most of the coupled fixed point theorems for single-valued mappings on ordered metric spaces are consequences of

well-known fixed point theorems. Many results appeared on multidimensional fixed point theory in different contexts including [2, 6-8, 13-20, 24, 25, 27-35, 38].
Let (X, d) be a metric space and $T:X\to X$ a self mapping. If (X, d) is complete and T is a contraction, that is, there exists a constant $k\in[0, 1)$ such that

$$d(Tx, Ty) \leq kd(x, y), \text{for all } x, y \in X,$$

then, by Banach contraction mapping principle, which is a classical and powerful tool in nonlinear analysis, we know that T has a unique fixed point p and, for any $x_0\in X$, the Picard iteration $\{T^n x_0\}$ converges to p. The Banach contraction mapping principle has been generalized in several directions, One of these generalizations, known as the Meir-Keeler fixed point theorem [26], has been obtained by replacing the contraction condition (1) by the following more general assumption: for all $\varepsilon>0$ there exists $\delta(\varepsilon)>0$ such that

$$x, y \in X, \varepsilon \leq d(x, y) < \varepsilon + \delta(\varepsilon) \Rightarrow d(Tx, Ty) < \varepsilon.$$

In [36], Samet established the coupled fixed points of mixed strict monotone generalized Meir-Keeler operators and also established the existence and uniqueness results for coupled fixed point. Berinde and Pecurar [3] obtained more general coupled fixed point theorems for mixed monotone operators $F:X\times X\to X$ satisfying a generalized Meir-Keeler contractive condition.
Our main aim in this manuscript is to obtain coincidence point result for g-non-decreasing mappings satisfying generalized Meir-Keeler contraction on ordered metric spaces. Using obtained result, we demonstrate the formation of coupled coincidence point result for generalized compatible pair of mappings. We obtain the solution of Fredholm nonlinear integral equation to indicate the usefulness of our result and also give an example. We generalize, modify, improve, sharpen and enrich the results of Berinde and Pecurar [3], Bhaskar and Lakshmikantham [4], Lakshmikantham and Ćirić [21], Meir and Keeler [26], Samet [36] and various well-known results of the existing literature.

2. Preliminaries

Suppose X is a non-empty set. For any natural number $n\geq 2$, let X^n be the nth Cartesian product $X\times X\times\ldots\times X$ (n times). If $g:X\to X$ is any self mapping, if $x\in X$, we shall denote g(x) by gx.
Definition 1 [10]. Let $F:X^2\to X$ be a given mapping. An element $(x, y)\in X^2$ is called a coupled fixed point of F if

$$F(x, y) = x \text{ and } F(y, x) = y.$$

Definition 2 [4]. Let $(X, \preccurlyeq)$ be a partially ordered set. Suppose $F:X^2\to X$ be a given mapping. We say that F has the mixed monotone property if for all $x, y\in X$, we have

$$x_1, x_2 \in X, x_1 \preccurlyeq x_2 \Rightarrow F(x_1, y) \preccurlyeq F(x_2, y),$$

and

$$y_1, y_2 \in X, y_1 \preccurlyeq y_2 \Rightarrow F(x, y_1) \succcurlyeq F(x, y_2).$$

Definition 3 [21]. Let $F:X^2\to X$ and $g:X\to X$ be given mappings. An element $(x, y)\in X^2$ is called a coupled coincidence point of the mappings F and g if

$$F(x,y) = gx \text{ and } F(y,x) = gy.$$

Definition 4 [21]. Let $F:X^2\to X$ and $g:X\to X$ be given mappings. An element $(x, y)\in X^2$ is called a common coupled fixed point of the mappings F and g if

$$x = F(x,y) = gx \text{ and } y = F(y,x) = gy.$$

Definition 5 [21]. The mappings $F:X^2\to X$ and $g:X\to X$ are said to be commutative if

$$gF(x,y) = F(gx,gy), \text{for all } (x,y) \in X^2.$$

Definition 6 [21]. Let $(X, \preccurlyeq)$ be a partially ordered set. Suppose $F:X^2\to X$ and $g:X\to X$ are given mappings. We say that F has the mixed g-monotone property if for all $x, y\in X$, we have

$$x_1, x_2 \in X, gx_1 \preccurlyeq gx_2 \Rightarrow F(x_1,y) \preccurlyeq F(x_2,y),$$

and

$$y_1, y_2 \in X, gy_1 \preccurlyeq gy_2 \Rightarrow F(x,y_1) \succcurlyeq F(x,y_2).$$

If g is the identity mapping on X, then F satisfies the mixed monotone property.

Definition 7 [5]. The mappings $F:X^2\to X$ and $g:X\to X$ are said to be compatible if

$$\lim_{n\to\infty} d\big(gF(x_n,y_n), F(gx_n,gy_n)\big) = 0$$
$$\lim_{n\to\infty} d(gF(y_n,x_n), F(gy_n,gx_n)) = 0,$$

whenever $\{x_n\}$ and $\{y_n\}$ are sequences in X such that

$$\lim_{n\to\infty} F(x_n,y_n) = \lim_{n\to\infty} gx_n = x,$$
$$\lim_{n\to\infty} F(y_n,x_n) = \lim_{n\to\infty} gy_n = y, \text{for some } x,y \in X.$$

Definition 8 [12]. Suppose that $F, G:X^2\to X$ are two mappings. F is said to be G-increasing with respect to $\preccurlyeq$ if for all $x, y, u, v\in X$, with $G(x, y)\preccurlyeq G(u, v)$ we have $F(x, y)\preccurlyeq F(u, v)$.

Definition 9 [12]. Let $F, G:X^2\to X$ be two mappings. We say that the pair $\{F, G\}$ is commuting if

$$F(G(x,y),G(y,x)) = G(F(x,y),F(y,x)), \text{for all } x,y \in X.$$

Definition 10 [12]. Suppose that $F, G:X^2\to X$ are two mappings. An element $(x, y)\in X^2$ is called a coupled coincidence point of mappings F and G if

$$F(x,y) = G(x,y) \text{ and } F(y,x) = G(y,x).$$

Definition 11 [12]. Let $(X, \preccurlyeq)$ be a partially ordered set, $F:X^2\to X$ and $g:X\to X$ are two mappings. We say that F is g-increasing with respect to $\preccurlyeq$ if for any $x, y\in X$,

$$gx_1 \preccurlyeq gx_2 \text{ implies } F(x_1,y) \preccurlyeq F(x_2,y),$$

and

$$gy_1 \preccurlyeq gy_2 \text{ implies } F(x,y_1) \preccurlyeq F(x,y_2).$$

Definition 12 [12]. Let $(X, \preccurlyeq)$ be a partially ordered set, $F:X^2\to X$ be a mapping. We say that F is increasing with respect to $\preccurlyeq$ if for any $x, y\in X$,

$$x_1 \preccurlyeq x_2 \text{ implies } F(x_1,y) \preccurlyeq F(x_2,y),$$

and

$$y_1 \preccurlyeq y_2 \text{ implies } F(x,y_1) \preccurlyeq F(x,y_2).$$

Definition 13 [12]. Let F, G:$X^2 \to X$ be two mappings. We say that the pair {F, G} is generalized compatible if

$$\lim_{n\to\infty} d(F(G(x_n, y_n), G(y_n, x_n)), G(F(x_n, y_n), F(y_n, x_n))) = 0$$
$$\lim_{n\to\infty} d(F(G(y_n, x_n), G(x_n, y_n)), G(F(y_n, x_n), F(x_n, y_n))) = 0,$$

whenever (x_n) and (y_n) are sequences in X such that

$$\lim_{n\to\infty} G(x_n, y_n) = \lim_{n\to\infty} F(x_n, y_n) = x,$$
$$\lim_{n\to\infty} G(y_n, x_n) = \lim_{n\to\infty} F(y_n, x_n) = y, \text{for some } x, y \in X.$$

Obviously, a commuting pair is a generalized compatible but not conversely in general.

Definition 14 [1, 9]. A coincidence point of two mappings T, g:$X \to X$ is a point $x \in X$ such that $Tx=gx$.

Definition 15 [8]. An ordered metric space (X, d, $\preccurlyeq$) is a metric space (X, d) provided with a partial order $\preccurlyeq$.

Definition 16 [4, 12]. An ordered metric space (X, d, $\preccurlyeq$) is said to be non-decreasing-regular (respectively, non-increasing-regular) if for every sequence $\{x_n\} \subseteq X$ such that $\{x_n\} \to x$ and $x_n \preccurlyeq x_{n+1}$ (respectively, $x_n \succcurlyeq x_{n+1}$) for all n, we have that $x_n \preccurlyeq x$ (respectively, $x_n \succcurlyeq x$) for all n. (X, d, $\preccurlyeq$) is said to be regular if it is both non-decreasing-regular and non-increasing-regular.

Definition 17 [8]. Let(X, $\preccurlyeq$) be a partially ordered set and let T, g:$X \to X$ be two mappings. We say that T is (g, $\preccurlyeq$)-non-decreasing if $Tx \preccurlyeq Ty$ for all x, $y \in X$ such that $gx \preccurlyeq gy$. If g is the identity mapping on X, we say that T is $\preccurlyeq$-non-decreasing.

Remark 18 [8]. If T is (g, $\preccurlyeq$)-non-decreasing and $gx=gy$, then $Tx=Ty$. It follows that

$$gx = gy \Rightarrow \{gx \preccurlyeq gy, gy \preccurlyeq gx\} \Rightarrow \{Tx \preccurlyeq Ty, Ty \preccurlyeq Tx\} \Rightarrow Tx = Ty.$$

Definition 19 [8]. Let (X, $\preccurlyeq$) be a partially ordered set and endow the product space X^2 with the following partial order:

$$(u, v) \sqsubseteq (x, y) \Leftrightarrow x \succcurlyeq u \text{ and } y \preccurlyeq v, \text{for all } (u, v), (x, y)\ X^2. \tag{1}$$

Definition 20 [5, 11, 23, 25]. Let (X, d, $\preccurlyeq$) be an ordered metric space. Two mappings T, g:$X \to X$ are said to be O-compatible if

$$\lim_{n\to\infty} d(gTx_n, Tgx_n) = 0,$$

provided that $\{x_n\}$ is a sequence in X such that $\{gx_n\}$ is $\preccurlyeq$-monotone, that is, it is either non-increasing or non-decreasing with respect to $\preccurlyeq$ and

$$\lim_{n\to\infty} Tx_n = \lim_{n\to\infty} gx_n \in X.$$

Lemma 21 [38]. Let (X, d) be a metric space. Suppose $Y=X^2$ and define Δ_n :$X^n \times X^n \to [0, +\infty)$, for $A=(a_1, a_2, ..., a_n)$, $B=(b_1, b_2, ..., b_n) \in X^n$, by

$$\Delta_n(A, B) = \frac{1}{n}\sum_{i=1}^{n} d(a_i, b_i). \tag{2}$$

Then Δ_n is metric on X^n and (X, d) is complete if and only if (X^n, Δ_n) is complete.

3. Main results

Theorem 22. Let $(X, d, \preccurlyeq)$ be an ordered metric space and let $T, g: X \to X$ be two mappings such that the following properties are fulfilled:

(i) $T(X) \subseteq g(X)$,
(ii) T is $(g, \preccurlyeq)$-non-decreasing,
(iii) there exists $x_0 \in X$ such that $gx_0 \preccurlyeq Tx_0$,
(iv) for each $\varepsilon > 0$, there exists $\delta(\varepsilon) > 0$ such that

$$\varepsilon \le d(gx, gy) \le \varepsilon + \delta(\varepsilon) \Rightarrow d(Tx, Ty) < \varepsilon,$$

for all $x, y \in X$ such that $gx \preccurlyeq gy$. Also assume that, at least, one of the following conditions holds.

(a) (X, d) is complete, T and g are continuous and the pair (T, g) is O-compatible,
(b) (X, d) is complete, T and g are continuous and commuting,
(c) $(g(X), d)$ is complete and $(X, d, \preccurlyeq)$ is non-decreasing-regular,
(d) (X, d) is complete, $g(X)$ is closed and $(X, d, \preccurlyeq)$ is non-decreasing-regular,
(e) (X, d) is complete, g is continuous and monotone-non-decreasing, the pair (T, g) is O-compatible and $(X, d, \preccurlyeq)$ is non-decreasing-regular.

Then T and g have, at least, a coincidence point.

Proof. We divide the proof into four steps.

Step 1. We claim that there exists a sequence $\{x_n\} \subseteq X$ such that $\{gx_n\}$ is $\preccurlyeq$-non-decreasing and $gx_{n+1} = Tx_n$, for all $n \ge 0$. Starting from $x_0 \in X$ given in (iii) and taking into account that $Tx_0 \in T(X) \subseteq g(X)$, there exists $x_1 \in X$ such that $Tx_0 = gx_1$. Then $gx_0 \preccurlyeq Tx_0 = gx_1$. Since T is $(g, \preccurlyeq)$-non-decreasing, $Tx_0 \preccurlyeq Tx_1$. Now $Tx_1 \in T(X) \subseteq g(X)$, so there exists $x_2 \in X$ such that $Tx_1 = gx_2$. Then $gx_1 = Tx_0 \preccurlyeq Tx_1 = gx_2$. Since T is $(g, \preccurlyeq)$-non-decreasing, $Tx_1 \preccurlyeq Tx_2$. Repeating this argument, there exists a sequence $\{x_n\}_{n \ge 0}$ such that $\{gx_n\}$ is $\preccurlyeq$-non-decreasing, $gx_{n+1} = Tx_n \preccurlyeq Tx_{n+1} = gx_{n+2}$ and

$$gx_{n+1} = Tx_n \text{ for all } n \ge 0. \tag{3}$$

Step 2. We claim that $\{d(gx_n, gx_{n-1})\} \to 0$. Now, by (24), for each $\varepsilon > 0$, there exists $\delta(\varepsilon) > 0$ such that

$$\varepsilon \le d(gx, gy) \le \varepsilon + \delta(\varepsilon) \Rightarrow d(Tx, Ty) < \varepsilon. \tag{4}$$

Condition (4) implies the strict contractive condition

$$d(Tx, Ty) < d(gx, gy), \tag{5}$$

for all $x, y \in X$ such that $gx \preccurlyeq gy$. Thus, by (5), we have

$$d(gx_{n+1}, gx_n) = d(Tx_n, Tx_{n-1}) < d(gx_n, gx_{n-1}),$$

which shows that the sequence of nonnegative numbers $\{\alpha_n\}_{n \ge 0}$ given by

$$\alpha_n = d(gx_n, gx_{n-1}), \tag{6}$$

is non-increasing. Therefore, there exists some $\varepsilon \ge 0$ such that

$$\lim_{n\to\infty} \alpha_n = \lim_{n\to\infty} d(gx_n, gx_{n-1}) = \varepsilon.$$

We shall prove that $\varepsilon = 0$. Suppose, to the contrary, that $\varepsilon > 0$. Then there exists a positive integer p such that

$$\varepsilon < \alpha_p < \varepsilon + \delta(\varepsilon) \Rightarrow d(Tx_p, Tx_{p-1}) < \varepsilon,$$

it follows, by (3), that

$$\alpha_{p+1} = d(gx_{p+1}, gx_p) < \varepsilon,$$

which is a contradiction. Thus $\varepsilon=0$ and hence

$$\lim_{n\to\infty} \alpha_n = \lim_{n\to\infty} d(gx_n, gx_{n-1}) = 0. \tag{7}$$

Step 3. We claim that $\{gx_n\}_{n\geq 0}$ is a Cauchy sequence in X. Let now $\varepsilon>0$ be arbitrary and $\delta(\varepsilon)$ the corresponding value from the hypothesis of our theorem. By (7), there exists a positive integer k such that

$$\alpha_{k+1} = d(gx_{k+1}, gx_k) < \delta(\varepsilon). \tag{8}$$

For this fixed number k, consider now the set $A_k=\{gx\in X: gx_k\preccurlyeq gx,\ d(gx_k, gx)<\varepsilon+\delta(\varepsilon)\}$. By (8), $A_k\neq\varphi$. We claim that

$$gx \in A_k \Rightarrow Tx \in A_k. \tag{9}$$

Let $gx\in A_k$. Then

$$d(gx_k, gx) < \varepsilon + \delta(\varepsilon) \Rightarrow d(Tx_k, Tx) < \varepsilon. \tag{10}$$

Now, by (3), (8) and (10), we have

$$\begin{aligned} d(gx_k, Tx) &\leq d(gx_k, Tx_k) + d(Tx_k, Tx) \\ &\leq d(gx_k, gx_{k+1}) + d(Tx_k, Tx) < \varepsilon + \delta(\varepsilon). \end{aligned}$$

Thus $Tx\in A_k$. Again

$$d(gx_k, gx_{k+1}) \leq d(gx_k, Tx) + d(Tx, gx_{k+1}) < 2(\varepsilon + \delta(\varepsilon)).$$

Thus $gx_{k+1}\in A_k$ and by induction,

$$gx_n \in A_k, \text{for all } n > k.$$

This implies that for all n, m>k, we have

$$d(gx_n, gx_m) \leq d(gx_n, gx_k) + d(gx_k, gx_m) < 2(\varepsilon + \delta(\varepsilon)) = 4\varepsilon.$$

This shows that $\{gx_n\}_{n\geq 0}$ is a Cauchy sequence in X.

Step 4. We claim that T and g have a coincidence point distinguishing between cases (a)-(e).

Suppose now that (a) holds, that is, (X, d) is complete, T and g are continuous and the pair (T, g) is O-compatible. Since (X, d) is complete, therefore there exists $z\in X$ such that $\{gx_n\}\to z$. Now $Tx_n=gx_{n+1}$ for all n, we also have that $\{Tx_n\}\to z$. As T and g are continuous, then $\{Tgx_n\}\to Tz$ and $\{ggx_n\}\to gz$. Since the pair (T, g) is O-compatible, we have $\lim_{n\to\infty} d(gTx_n, Tgx_n)=0$. In such a case, we conclude that $d(gz, Tz)=\lim_{n\to\infty} d(ggx_{n+1}, Tgx_n)=\lim_{n\to\infty} d(gTx_n, Tgx_n)=0$, that is, z is a coincidence point of T and g.

Suppose now that (b) holds, that is, (X, d) is complete, T and g are continuous and commuting. Clearly (b) implies (a).

Suppose now that (c) holds, that is, (g(X), d) is complete and $(X, d, \preccurlyeq)$ is non-decreasing-regular. As $\{gx_n\}$ is a Cauchy sequence in the complete space (g(X), d), so there exist $y\in g(X)$ such that $\{gx_n\}\to y$. Let $z\in X$ be any point such that $y=gz$. In this case $\{gx_n\}\to gz$. Indeed, as $(X, d, \preccurlyeq)$ is non-decreasing-regular and $\{gx_n\}$ is $\preccurlyeq$-non-decreasing and converging to gz, we deduce that $gx_n\preccurlyeq gz$ for all $n\geq 0$. Applying the contractive condition (iv), $d(gx_{n+1}, Tz)<d(Tx_n, Tz)<d(gx_n, gz)$. Taking $n\to\infty$, we get $d(gz, Tz)=0$, that is, z is a coincidence point of T and g.

Suppose now that (d) holds, that is, (X, d) is complete, g(X) is closed and $(X, d, \preccurlyeq)$ is non-decreasing-regular. It follows from the fact that a closed subset of a complete metric space is also complete. Then, $(g(X), d)$ is complete and $(X, d, \preccurlyeq)$ is non-decreasing-regular. Clearly (d) implies (c).

Suppose now that (e) holds, that is, (X, d) is complete, g is continuous and monotone-non-decreasing, the pair (T, g) is O-compatible and $(X, d, \preccurlyeq)$ is non-decreasing-regular. As (X, d) is complete, so there exists $z\in X$ such that $\{gx_n\}\to z$. Since $Tx_n=gx_{n+1}$ for all n, we also have that $\{Tx_n\}\to z$. As T and g are continuous, then $\{Tgx_n\}\to Tz$ and $\{ggx_n\}\to gz$. Since the pair (T, g) is O-compatible, therefore $\lim_{n\to\infty}d(gTx_n, Tgx_n)=0$. Thus, we conclude that $d(gz, Tz)=\lim_{n\to\infty}d(ggx_{n+1}, Tgx_n)=\lim_{n\to\infty}d(gTx_n, Tgx_n)=0$, that is, z is a coincidence point of T and g. Again, as $(X, d, \preccurlyeq)$ is non-decreasing-regular and $\{gx_n\}$ is $\preccurlyeq$-non-decreasing and converging to gz, we deduce that $gx_n\preccurlyeq gz$ for all $n\geq 0$. Applying the contractive condition (iv), $d(gx_{n+1}, Tz)<d(Tx_n, Tz)<d(gx_n, gz)$. Taking $n\to\infty$, we get $d(gz, Tz)=0$, that is, z is a coincidence point of T and g.

Next, we formulate the coupled version of Theorem 22. Consider the ordered metric space $(X^2, \Delta_2, \sqsubseteq)$, where Δ_2 was defined in Lemma 21 and $\sqsubseteq$ was introduced in (1). Define the mappings $T_F, T_G:X^2\to X^2$, for all $(x, y)\in X^2$, by

$$T_F(x,y) = (F(x,y), F(y,x)) \text{ and } T_G(x,y) = (G(x,y), G(y,x)).$$

Under these conditions, the following properties hold:

Lemma 23. Let $(X, d, \preccurlyeq)$ be an ordered metric space and let $F, G:X^2\to X$ be two mappings. Then

(1) (X, d) is complete if and only if (X^2, Δ_2) is complete.

(2) If $(X, d, \preccurlyeq)$ is regular, then $(X^2, \Delta_2, \sqsubseteq)$ is also regular.

(3) If F is d-continuous, then T_F is Δ_2-continuous.

(4) If F is G-increasing with respect to $\preccurlyeq$, then T_F is $(T_G, \sqsubseteq)$-non-decreasing.

(5) If there exist two elements $x_0, y_0\in X$ with $G(x_0, y_0)\preccurlyeq F(x_0, y_0)$ and $G(y_0, x_0)\succcurlyeq F(y_0, x_0)$, then there exists a point $(x_0, y_0)\in X^2$ such that $T_G(x_0, y_0)\sqsubseteq T_F(x_0, y_0)$.

(6) For any $x, y\in X$, there exist $u, v\in X$ such that $F(x, y)=G(u, v)$ and $F(y, x)=G(v, u)$, then $T_F(X^2)\subseteq T_G(X^2)$.

(7) For each $\varepsilon>0$, there exists $\delta(\varepsilon)>0$ such that

$$\varepsilon \leq \frac{d(G(x,y), G(u,v)) + d(G(y,x), G(v,u))}{2} \leq \varepsilon + \delta(\varepsilon),$$

implies

$$\frac{d(F(x,y), F(u,v)) + d(F(y,x), F(v,u))}{2} < \varepsilon,$$

for all $x, y, u, v\in X$, where $G(x, y)\preccurlyeq G(u, v)$ and $G(y, x)\succcurlyeq G(v, u)$, then

$$\varepsilon \leq \Delta_2(T_G(x,y), T_G(u,v)) \leq \varepsilon + \delta(\varepsilon) \Rightarrow \Delta_2(T_F(x,y), T_F(u,v)) < \varepsilon,$$

for all $(x, y), (u, v)\in X^2$, where $T_G(x, y)\sqsubseteq T_G(u, v)$.

(8) If the pair $\{F, G\}$ is generalized compatible, then the mappings T_F and T_G are O-compatible in $(X^2, \Delta_2, \sqsubseteq)$.

(9) A point $(x, y)\in X^2$ is a coupled coincidence point of F and G if and only if it is a coincidence point of T_F and T_G.

Proof. Item (1) follows from Lemma 21 and items (2), (3), (5), (6) and (9) are obvious.

(4) Assume that F is G-increasing with respect to $\preccurlyeq$ and let (x, y), (u, v)$\in X^2$ be such that $T_G(x, y)\sqsubseteq T_G(u, v)$. Then $G(x, y)\preccurlyeq G(u, v)$ and $G(y, x)\succcurlyeq G(v, u)$. Since F is G-increasing with respect to $\preccurlyeq$, therefore $F(x, y)\preccurlyeq F(u, v)$ and $F(y, x)\succcurlyeq F(v, u)$. So $T_F(x, y)\sqsubseteq T_F(u, v)$, which means that T_F is $(T_G, \sqsubseteq)$-non-decreasing.

(7) For each $\varepsilon>0$, there exists $\delta(\varepsilon)>0$ such that

$$\varepsilon \le \frac{d(G(x,y),G(u,v)) + d(G(y,x),G(v,u))}{2} \le \varepsilon + \delta(\varepsilon),$$

implies

$$\frac{d(F(x,y),F(u,v))+d(F(y,x),F(v,u))}{2} < \varepsilon \tag{11}$$

for all x, y, u, v$\in$X, where $G(x, y)\preccurlyeq G(u, v)$ and $G(y, x)\succcurlyeq G(v, u)$ and let (x, y), (u, v)$\in X^2$ be such that $T_G(x, y)\sqsubseteq T_G(u, v)$ and $\varepsilon\le\Delta_2(T_G(x, y), T_G(u, v))\le\varepsilon+\delta(\varepsilon)$. Therefore $G(x, y)\preccurlyeq G(u, v)$ and $G(y, x)\succcurlyeq G(v, u)$ and

$$\varepsilon \le \frac{d(G(x,y),G(u,v)) + d(G(y,x),G(v,u))}{2} \le \varepsilon + \delta(\varepsilon),$$

Using (11), we have

$$\frac{d(F(x,y),F(u,v))+d(F(y,x),F(v,u))}{2} < \varepsilon, \tag{12}$$

Thus, it follows from (12) that

$$\begin{aligned} &\Delta_2(TF(x,y),TF(u,v)) \\ &= \Delta_2\Big(\big(F(x,y),F(y,x)\big),\big(F(u,v),F(v,u)\big)\Big) \\ &= \frac{d(F(x,y),F(u,v)) + d(F(y,x),F(v,u))}{2} < \varepsilon, \end{aligned}$$

(8) Let $\{(x_n, y_n)\}\subseteq X^2$ be any sequence such that $T_F(x_n, y_n)\to(x, y)$ and $T_G(x_n, y_n)\to(x, y)$ (notice that we do not need to suppose that $\{T_G(x_n, y_n)\}$ is $\sqsubseteq$-monotone). Therefore,

$$(F(x_n,y_n),F(y_n,x_n)) \to (x,y) \Rightarrow F(x_n,y_n) \to x \text{ and } F(y_n,x_n) \to y,$$

and

$$(G(x_n,y_n),G(y_n,x_n) \to (x,y) \Rightarrow G(x_n,y_n) \to x \text{ and } G(y_n,x_n) \to y.$$

Therefore

$$\lim_{n\to\infty} F(x_n,y_n) = \lim_{n\to\infty} G(x_n,y_n) = x \in X,$$
$$\lim_{n\to\infty} F(y_n,x_n) = \lim_{n\to\infty} G(y_n,x_n) = y \in X.$$

Since the pair {F, G} is generalized compatible, we have

$$\lim_{n\to\infty} d(F(G(x_n,y_n),G(y_n,x_n)),G(F(x_n,y_n),F(y_n,x_n))) = 0,$$
$$\lim_{n\to\infty} d(F(G(y_n,x_n),G(x_n,y_n)),G(F(y_n,x_n),F(x_n,y_n))) = 0.$$

In particular

$$\begin{aligned} &\lim_{n\to\infty} \Delta_2(T_G T_F(x_n,y_n),T_F T_G(x_n,y_n)) \\ &= \lim_{n\to\infty} \Delta_2(T_G(F(x_n,y_n),F(y_n,x_n)),T_F(G(x_n,y_n),G(yn,xn))) \\ &= \lim_{n\to\infty} \Delta_2\begin{pmatrix} ((G(F(x_n,y_n),F(y_n,x_n)),G(F(y_n,x_n),F(x_n,y_n))), \\ (F(G(x_n,y_n),G(y_n,x_n)),F(G(y_n,x_n),G(x_n,y_n)))) \end{pmatrix} \end{aligned}$$

$$= \lim_{n\to\infty}\left(\frac{1}{2}\right)[d(G(F(x_n,y_n),F(y_n,x_n)),F(G(x_n,y_n),G(y_n,x_n)))$$
$$+d(G(F(y_n,x_n),F(x_n,y_n)),F(G(y_n,x_n),G(x_n,y_n)))] = 0.$$

Hence, the mappings T_F and T_G are O-compatible in $(X^2, \Delta_2, \sqsubseteq)$.

Theorem 24. Let $(X, \preccurlyeq)$ be a partially ordered set such that there exists a complete metric d on X. Assume F, G:X×X→X be two generalized compatible mappings satisfying (11) such that F is G-increasing with respect to $\preccurlyeq$, G is continuous and has the mixed monotone property and there exist two elements x_0, $y_0 \in X$ with

$$G(x_0,y_0) \preccurlyeq F(x_0,y_0) \text{ and } G(y_0,x_0) \succcurlyeq F(y_0,x_0).$$

Suppose that for any x, y∈X, there exist u, v∈X such that

$$F(x,y) = G(u,v) \text{ and } F(y,x) = G(v,u). \quad (13)$$

Also suppose that either

(a) F is continuous or

(b) $(X, d, \preccurlyeq)$ is regular.

Then F and G have a coupled coincidence point.

Proof. It is only require to use Theorem 22 to the mappings $T=T_F$ and $g=T_G$ in the ordered metric space $(X^2, \Delta_2, \sqsubseteq)$ by applying Lemma 21.

Corollary 25. Let $(X, \preccurlyeq)$ be a partially ordered set such that there exists a complete metric d on X. Assume F, G:X×X→X be two commuting mappings satisfying (11) and (13) such that F is G-increasing with respect to $\preccurlyeq$, G is continuous and has the mixed monotone property and there exist two elements x_0, $y_0 \in X$ with

$$G(x_0, y_0) \preccurlyeq F(x_0, y_0) \text{ and } G(y_0, x_0) \succcurlyeq F(y_0, x_0).$$

Also suppose that either

(a) F is continuous or

(b) $(X, d, \preccurlyeq)$ is regular.

Then F and G have a coupled coincidence point.

Now we obtain the results without mixed g-monotone property of F.

Corollary 26. Let $(X, \preccurlyeq)$ be a partially ordered set such that there exists a complete metric d on X. Assume $F:X^2\to X$ and g:X→X be two mappings such that F is g-increasing with respect to $\preccurlyeq$ and for each ε>0, there exists δ(ε)>0 such that

$$\varepsilon \le \left(\frac{d(gx,gu)+d(gy,gv)}{2}\right) \le \varepsilon + \delta(\varepsilon),$$

implies

$$\frac{d(F(x,y),F(u,v))+d(F(y,x),F(v,u))}{2} < \varepsilon. \quad (14)$$

for all x, y, u, v∈X, where $g(x)\preccurlyeq g(u)$ and $g(y)\succcurlyeq g(v)$. Suppose that $F(X^2)\subseteq g(X)$, g is continuous and monotone increasing with respect to $\preccurlyeq$ and the pair {F, g} is compatible. Also suppose that either

(a) F is continuous or

(b) $(X, d, \preccurlyeq)$ is regular.

If there exist two elements x_0, $y_0 \in X$ with

$$gx_0 \preccurlyeq F(x_0,y_0) \text{ and } gy_0 \succcurlyeq F(y_0,x_0).$$

Then F and g have a coupled coincidence point.

Corollary 27. Let $(X, \preccurlyeq)$ be a partially ordered set such that there exists a complete metric d on X. Assume $F:X^2\to X$ and $g:X\to X$ be two mappings such that F is g-increasing with respect to $\preccurlyeq$ and satisfying (14). Suppose that $F(X^2)\subseteq g(X)$, g is continuous and monotone increasing with respect to $\preccurlyeq$ and the pair {F, g} is commuting. Also suppose that either

(a) F is continuous or

(b) $(X, d, \preccurlyeq)$ is regular.

If there exist two elements $x_0, y_0\in X$ with

$$gx_0 \preccurlyeq F(x_0, y_0) \text{ and } gy_0 \succcurlyeq F(y_0, x_0).$$

Then F and g have a coupled coincidence point.

Now, we derive the result without mixed monotone property of F.

Corollary 28. Let $(X, \preccurlyeq)$ be a partially ordered set such that there exists a complete metric d on X. Assume $F:X^2\to X$ be an increasing mapping with respect to $\preccurlyeq$ and for each $\varepsilon>0$, there exists $\delta(\varepsilon)>0$ such that

$$\varepsilon \leq \left(\frac{d(x, u) + d(y, v)}{2}\right) \leq \varepsilon + \delta(\varepsilon),$$

implies

$$\frac{d\big(F(x, y), F(u, v)\big) + d\big(F(y, x), F(v, u)\big)}{2} < \varepsilon. \tag{15}$$

for all x, y, u, $v\in X$, where $x\preccurlyeq u$ and $y\succcurlyeq v$. Also suppose that either

(a) F is continuous or

(b) $(X, d, \preccurlyeq)$ is regular.

If there exist two elements $x_0, y_0\in X$ with

$$x_0 \preccurlyeq F(x_0, y_0) \text{ and } y_0 \succcurlyeq F(y_0, x_0).$$

Then F has a coupled fixed point.

Example 29. Suppose $X=\mathbb{R}$, furnished with the usual metric $d:X\times X\to[0, +\infty)$ with the natural ordering of real numbers $\leq$. Let $F, G:X\times X\to X$ be defined as

$$F(x, y) = \begin{cases} \dfrac{x^2 - y^2}{3}, \text{if } x \geq y, \\ 0, \text{if } x < y, \end{cases}$$

and

$$G(x, y) = \begin{cases} x^2 - y^2, \text{if } x \geq y, \\ 0, \text{if } x < y, \end{cases}$$

First, we shall show that the mappings F and G satisfy the contractive condition of Theorem 24. Let x, y, u, $v\in X$ such that $G(x, y)\preccurlyeq G(u, v)$ and $G(y, x)\succcurlyeq G(v, u)$, such that

$$\varepsilon \leq \frac{d\big(G(x, y), G(u, v)\big) + d\big(G(y, x), G(v, u)\big)}{2} \leq \varepsilon + \delta(\varepsilon),$$

that is,

$$\varepsilon \le \left(\frac{1}{2}\right)[|x^2 - y^2| + |u^2 - v^2|] \le \varepsilon + \delta(\varepsilon).$$

Then

$$\left(\frac{d\big(F(x,y),F(u,v)\big) + d\big(F(y,x),F(v,u)\big)}{2}\right)$$
$$= \left(\frac{1}{2}\right)\left[\left|\left(\frac{x^2-y^2}{3}\right) - \left(\frac{u^2-v^2}{3}\right)\right| + \left|\left(\frac{y^2-x^2}{3}\right) - \left(\frac{v^2-u^2}{3}\right)\right.\right.$$
$$= \left|\left(\frac{x^2-y^2}{3}\right) - \left(\frac{u^2-v^2}{3}\right)\right|$$
$$\le \left(\frac{1}{3}\right)[|x^2 - y^2| + |u^2 - v^2|]$$
$$\le \left(\frac{2}{3}\right)\big(\varepsilon + \delta(\varepsilon)\big) < \varepsilon.$$

Thus the contractive condition of Theorem 24 is satisfied for all x, y, u, v∈X. Moreover, like in [12], all the other conditions of Theorem 24 are satisfied and {F, G} have a coincidence point z=(0, 0).

4. Application to integral equations

As an application of the results constructed in previous section, we study the existence of the solution to a Fredholm nonlinear integral equation. We shall consider the following integral equation

$$X(p) = \int_a^b \big(K^1(p,q) + K^2(p,q)\big)\big[f\big(q,x(q)\big) + g\big(q,x(q)\big)\big]dq + \quad h(p), \quad (16)$$

for all p∈I=[a, b].

Let Θ denote the set of all functions θ:[0, +∞)→[0, +∞) satisfying

(i_θ) θ is non-decreasing,

(ii_θ) θ(p)≤p.

Assumption 30. We assume that the functions K_1, K_2, f, g fulfill the following conditions:

(i) $K_1(p, q)\ge 0$ and $K_2(p, q)\le 0$ for all p, q∈I.

(ii) There exist the positive numbers λ, μ and θ∈Θ such that for all x, y∈ ℝ with x≽y, the following conditions hold:

$$0 \le f(q,x) - f(q,y) \le \lambda\theta(x-y), \quad (17)$$

$$-\mu\theta(x-y) \le g(q,x) - g(q,y) \le 0. \quad (18)$$

(iii) $\max\{\lambda, \mu\}\sup_{p\in I}\int_a^b (K_1(p,q) - K_2(p,q))\,dq \le (1/6).$ (19)

Definition 31 [22]. A pair (α, β)∈X² with X=C(I, ℝ), where C(I, ℝ) denote the set of all continuous functions from I to ℝ, is called a coupled lower-upper solution of (16) if, for all p∈I,

$$\alpha(p) \leq \int_a^b K_1(p,q)[f(q,\alpha(q)) + g(q,\beta(q))]dq$$
$$+\int_a^b K_1(p,q)[f(q,\beta(q)) + g(q,\alpha(q))]dq + h(p),$$
and
$$\beta(p) \geq \int_a^b K_1(p,q)[f(q,\beta(q)) + g(q,\alpha(q))]dq$$
$$+\int_a^b K_1(p,q)[f(q,\alpha(q)) + g(q,\beta(q))]dq + h(p)$$

Theorem 32. Consider the integral equation (16) with K_1, $K_2 \in C(I\times I, \mathbb{R})$, f, $g\in C(I\times\mathbb{R}, \mathbb{R})$ and $h\in C(I, \mathbb{R})$. Suppose that there exists a coupled lower-upper solution (α, β) of (16) and Assumption 30 is satisfied. Then the integral equation (16) has a solution in $C(I, \mathbb{R})$.

Proof. Consider $X=C(I, \mathbb{R})$, the natural partial order relation, that is, for x, $y\in C(I, \mathbb{R})$,
$$x \preccurlyeq y \Leftrightarrow x(p) \leq y(p), \forall p \in I.$$
Obviously X is a complete metric space with respect to the sup metric
$$d(x,y) = \sup_{p\in I}|x(p) - y(p)|.$$
Now take the following partial order on X×X: for (x, y), (u, v)∈X×X,
$$(x,y) \preccurlyeq (u,v) \Leftrightarrow x(p) \leq u(p) \text{ and } y(p) \geq v(p), \forall p \in I.$$
Define the mapping $F:X\times X\to X$, for all p∈I, by
$$F(x,y)(p) = \int_a^b K_1(p,q)[f(q,x(q)) + g(q,y(q))]dq$$
$$+\int_a^b K_1(p,q)[f(q,y(q)) + g(q,x(q))]dq + h(p).$$
It is easy to see, like in [12], that F is increasing. Let x, y, u, v∈X with $x\succcurlyeq u$ and $y\preccurlyeq v$, such that
$$\varepsilon \leq \left(\frac{d(x,u)+d(y,v)}{2}\right) \leq \varepsilon + \delta(\varepsilon). \tag{20}$$
Now
$$F(x,y)(p) - F(u,v)(p)$$
$$=\int_a^b K^1(p,q)\left[\Big(f\big(q,x(q)\big) - f\big(q,u(q)\big)\Big) - \Big(g\big(q,v(q)\big) - g\big(q,y(q)\big)\Big)\right]dq$$
$$-\int_a^b K_2(p,q)[(f(q,v(q)) - f(q,y(q))) - (g(q,x(q)) - g(q,u(q)))]dq.$$
Thus, by using (17) and (18), we have
$$F(x,y)(p) - F(u,v)(p)$$
$$\leq \int_a^b K_1(p,q)[\lambda\theta(x(q) - u(q)) + \mu\theta(v(q) - y(q))]dq$$
$$-\int_a^b K_2(p,q)[\lambda\theta(v(q) - y(q)) + \mu\theta(x(q) - u(q))]dq. \tag{21}$$
Since the function θ is non-decreasing and $x\succcurlyeq u$, $y\preccurlyeq v$, we have
$$\theta(x(q) - u(q)) \leq \theta(\sup_{q\in I}|x(q) - u(q)|) = \theta(d(x,u)),$$
$$\theta(v(q) - y(q)) \leq \theta(\sup_{q\in I}|v(q) - y(q)|) = \theta(d(y,v)).$$
Hence by (21), in view of the fact that $K_2(p, q)\leq 0$, we obtain

$$|F(x,y)(p) - F(u,v)(p)|$$
$$\leq \int_a^b K_1(p,q)[\lambda\theta(d(x,u)) + \mu\theta(d(y,v))]dq$$
$$-\int_a^b K_2(p,q)[\lambda\theta(d(y,v)) + \mu\theta(d(x,u))]dq$$
$$\leq \int_a^b K_1(p,q)[\max\{\lambda,\mu\}\theta(d(x,u)) + \max\{\lambda,\mu\}\theta(d(y,v))]dq$$
$$-\int_a^b K_2(p,q)[\max\{\lambda,\mu\}\theta(d(y,v)) + \max\{\lambda,\mu\}\theta(d(x,u))]dq,$$

as all the quantities on the right hand side of (21) are non-negative. Now, taking the supremum with respect to p, by using (19), we get

$$d(F(x,y),F(u,v))$$
$$\leq \max\{\lambda,\mu\}\sup_{p\in I}\int_a^b (K_1(p,q) - K_2(p,q))dq.\left[\theta\left(d(x,u)\right) + \theta\left(d(y,v)\right)\right]$$
$$\leq \frac{\theta(d(x,u))+\theta(d(y,v))}{6}.$$

Thus

$$d(F(x,y),F(u,v)) \leq \frac{\theta(d(x,u))+\theta(d(y,v))}{6}.$$

Similarly

$$d(F(y,x),F(v,u)) \leq \frac{\theta(d(x,u))+\theta(d(y,v))}{6}.$$

Combining them, we get

$$\frac{d(F(x,y),F(u,v)) + d(F(y,x),F(v,u))}{2}$$
$$\leq \frac{\theta(d(x,u))+\theta(d(y,v))}{6}. \tag{22}$$

Now, since θ is non-decreasing, we have

$$\theta(d(x,u)) \leq \theta(d(x,u) + d(y,v)),$$
$$\theta(d(y,v)) \leq \theta(d(x,u) + d(y,v)),$$

which implies, by (ii_θ), that

$$\frac{\theta(d(x,u)) + \theta(d(y,v))}{2}$$
$$\leq \theta(d(x,u) + d(y,v)) \leq d(x,u) + d(y,v).$$

Hence

$$\frac{\Theta\left(d(x,u)\right)+\theta\left(d(y,v)\right)}{6} \leq \left(\frac{1}{3}\right)[d(x,u) + d(y,v)] \tag{23}$$

Thus by (20), (22) and (23), we have

$$\frac{d\left(F(x,y),F(u,v)\right) + d\left(F(y,x),F(v,u)\right)}{2}$$
$$\leq \left(\frac{1}{3}\right)[d(x,u) + d(y,v)]$$
$$\leq \left(\frac{2}{3}\right)\left(\varepsilon + \delta(\varepsilon)\right) < \varepsilon,$$

which is the contractive condition (15) of Corollary 30. Now, let (α, β)∈X×X be a coupled upper-lower solution of (16), then we have α(p)≤F(α, β)(p) and β(p)≥F(β, α)(p), for all p∈I, which shows that all hypothesis of Corollary 30 are satisfied.

This proves that F has a coupled fixed point $(x, y)\in X\times X$ which is the solution in $X=C(I, \mathbb{R})$ of the integral equation (16).

Remark 33. Using the same criterion that can be used in [18-20, 25, 35, 37] we can obtain tripled, quadruple and in general, multidimensional coincidence point theorems from Theorem 24.

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Использование обобщенного сокращения Меира-Киллера для результатов точки совпадения на упорядоченных метрических пространствах с применением

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РЕЗЮМЕ

Приведем теорему о совпадении точек для g-неубывающих отображений, удовлетворяющих обобщенному сокращению Меира-Киллера на упорядоченных метрических пространствах. Используя полученный результат, мы продемонстрируем формирование связанного результата точки совпадения для обобщенной совместимой пары отображений. Получаем решение нелинейного интегрального уравнения Фредгольма, чтобы показать полезность нашего результата, а также привести пример. Наши результаты обобщают, изменяют, улучшают и обогащают несколько известных результатов.

Ключевые слова: Точка совпадения, в сочетании точки совпадение, обобщенное сокращение Меира-Киллера, упорядоченное метрическое пространство, O-совместимая, обобщенная совместимость, g-неубывающее отображение, смешанное монотонное отображение, коммутирующее отображение.